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# Forecasting Volatility of Cryptocurrencies: The Role of GARCH-Family Models

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**Resumo**
This paper investigates the ability of GARCH-family models to estimate and forecast the volatility process of twelve leading cryptocurrencies: Binance Coin, Bitcoin Cash, Bitcoin, Cardano, Chainlink, Dogecoin, Ethereum Classic, Ethereum, Litecoin, Ripple, Stellar and Vechain. The aim is to test whether or not GARCH models provide risk managers more accurate volatility forecasts for digital currencies as high volatilile assets. A large empirical analysis is performed by comparing one-step-ahead volatility predictions from different specifications of GARCH models. GARCH specifications are up to four scedastic functions and six error distributions, resulting in a total of twenty four estimated models for each cryptocurrency studied in this work. Results showed that component GARCH model is the structure that achieved higher accuracy in modeling and forecasting the volatility process of the digital currencies returns. It also indicates that, in such market, the conditional variance dynamics is better described by a permanent and a transitory components, allowing for transitory shocks.

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**Abstract**

This paper investigates the ability of GARCH-family models to estimate and forecast the volatility process of twelve leading cryptocurrencies: Binance Coin, Bitcoin Cash, Bitcoin, Cardano, Chainlink, Dogecoin, Ethereum Classic, Ethereum, Litecoin, Ripple, Stellar and Vechain. The aim is to test whether or not GARCH models provide risk managers more accurate volatility forecasts for digital currencies as high volatilile assets. A large empirical analysis is performed by comparing one-step-ahead volatility predictions from different specifications of GARCH models. GARCH specifications are up to four scedastic functions and six error distributions, resulting in a total of twenty four estimated models for each cryptocurrency studied in this work. Results showed that component GARCH model is the structure that achieved higher accuracy in modeling and forecasting the volatility process of the digital currencies returns. It also indicates that, in such market, the conditional variance dynamics is better described by a permanent and a transitory components, allowing for transitory shocks.

**Keywords:** volatility, forecasting, cryptocurrency, GARCH models.

# 1 Introduction

Since the publication of the white paper on the first functional digital currency, Bitcoin (Nakamoto, 2008), the importance of cryptocurrencies and their derivative products in the global financial market has increased significantly. In October 2021, the capitalization of Bitcoin was already more than USD 1 trillion and the Ethereum Coin was in second place with a capitalization value of more than USD 500 billion[1]. This capitalization was also accompanied by large price fluctuations. In 2021 alone, Bitcoin's return was 59.8%, while Dogecoin's return was 3,536.0%.

If on the one hand this growing interest in digital currencies grabbed the attention of authorities and central banks around the world, as it escapes the monetary control of individual countries, on the other hand it encourages the emergence of new financial products and investment opportunities.

In 2016, BitMEX exchange introduced perpetual futures contracts (i.e. perpetual swaps), an instrument with no expiration date, so that the position can be held as long as the buyer or seller wishes. In December 2017, the Chicago Mercantile Exchange (CME) and the Chicago Board Options Exchange (CBOE) began trading Bitcoin futures contracts, adding weight to the cryptocurrency derivatives market. In October 2021, the ProShares company launched the first American futures exchange-traded fund (ETF) linked to Bitcoin (ticker: BITO) and the cryptocurrency market. In this context, notice that the ETF market tends to be significantly more volatile than the traditional market, absorbing the high volatility with which the cryptocurrency market is marked[2].

In addition, the options market, which uses crypto assets as underlying assets, has also been developing. In October 2017, the Commodity Futures Trading Commission granted the exchange LedgerX a license to trade Eu-

[1]Source: www.coinmarketcap.com. Access on 4, February, 2022.

[2]Source: https://www.coindesk.com/markets/2021/01/13/trading-hall-of-fame-the-bitcoin-options-bet-that-made-582m-profit-on-just-638k/. Access on 11, March, 2022.

ropean call and put options contracts. Meanwhile, Coindesk and Deribit, other exchanges trading digital currencies, also started trading cryptocurrency options (Söylemez, 2019). In October 2021, the daily volume of traded derivatives already exceeded USD 200 billion. In this sense, the growing cryptocurrency options market is characterized by periods of higher volatility compared to the traditional market. As in the spot market, large returns and losses have accompanied these new derivatives of the cryptocurrency market[3]. This has been leading the academic literature to increasingly look at models that better measure volatility, the main variable in option pricing (see Black and Scholes).

The development of the digital currency market and the creation of new derivatives financial products have also been accompanied by other important developments. In September 2021, El Salvador adopted Bitcoin as legal tender alongside the U.S. dollar. The country's president, Nayib Bukele, defended the introduction of the cryptocurrency as a way to boost their economy, create new jobs and promote financial inclusion. As part of the strategy to promote the use of Bitcoin, the government has developed a mobile application, Chivo Wallet, which provides 30 USD in Bitcoin to each resident who adopts the digital wallet. In addition, he guaranteed $150 million to ensure the free convertibility of Bitcoin to the dollar. On the other hand, this action is still viewed with skepticism by other countries and international bodies. In June 2021, the International Monetary Fund (IMF) warned of potential problems arising from the adoption of cryptocurrencies as official currencies, which could affect future negotiations between the government that performed such an act and the IMF[4].

The advance of the cryptocurrency market has also led to changes in the structures of central banks around the world, which are concerned about the

[3]Source: https://www.coindesk.com/markets/2021/01/13/trading-hall-of-fame-the-bitcoin-options-bet-that-made-582m-profit-on-just-638k/. Access on 11, March, 2022.

[4]Source: www.bloomberg.com/news/articles/2021-10-19/bitcoin-is-part-of-the-pitch-as-el-salvador-courts-imf-deal. Access on 11, March, 2022.

risk that decentralized digital currencies may pose to the control of their respective economic and financial structures. In April 2020, the People's Bank of China (PBOC) began testing their Central Bank Digital Currencies (CBDCs), a blockchain- based technology. CBDCs are a digital form of a fiat currency and feature great flexibility and ease of use. According to the Bank of England, CBDCs have the following characteristics: 1) more accessible than traditional reserves; 2) greater retail functionality than paper money; 3) separate structure from other forms of central bank money, which allows them to be used for different core purposes; 4) can earn interest at different rates than reserves do (Agur et al., 2022; Kumhof and Noone, 2018; Duffie et al., 2021). The European Central Bank has also shown interest in developing a digital euro, as has the Federal Reserve, which wants to create a digital dollar[5].

With this boom of new financial products being introduced within the digital currency market comes a growing speculative activity among participants. The number of players interested in realizing the high returns associated with cryptocurrency appreciation has increased significantly, marking a speculative motivation (Gregoriou, 2019; Corbet et al., 2018a; Baek and Elbeck, 2015; Glaser et al., 2014). Cheah and Fry (2015) pointed out that the Bitcoin market is prone to speculative bubbles that are substantial and unrelated to fundamental asset values. Similar results are also observed by Chaim and Laurini (2019), Corbet et al. (2018b) and Blau (2017).

Due to the great complexity projected by this scenario, the high volatility exhibited by crypto assets and their derivative products, the large number and diversity of investors presenting themselves, and the capitalization of a market that already exceeds $2.5 trillion[6], it is necessary to properly model price variability in the cryptocurrency market. Modeling and predicting the

[5]Source: https://www.americanbanker.com/payments/news/why-the-u-s-and-europe-trail-china-in-central-bank-currency-race. Access on 11, March, 2022.

[6]Source: www.finance.yahoo.com/news/total-crypto-market-cap-hits-062333457.html. Access on 11, March, 2022.

volatility of digital currency price returns appears to be a fundamental factor in setting investment strategies and managing risk, especially in a market that regularly observes periods of significant appreciation and depreciation.

In the literature dealing with volatility modeling and prediction, traditional assets such as stocks, indices, foreign exchange and commodities generally consider the use of models from the GARCH (Generalized Autoregressive Conditional Heteroskedasticity) family, a method that provides robust results compared to other techniques(Alberg et al., 2008; Marcucci, 2005; Garcia et al., 2005). There are still few works that comprehensively evaluate GARCH family models for cryptocurrencies, with most of them focusing on Bitcoin solely (Katsiampa, 2017; Cerqueti et al., 2020; Chi and Hao, 2020; Venter et al., 2020; Fakhfekh and Jeribi, 2020).

Given these findings and the evidence of high volatility in cryptocurrency markets, the selection of appropriate risk modeling approaches plays a crucial role in decision making when digital currencies are traded. This paper aims to evaluate GARCH-family models to estimate and forecast the volatility process of twelve leading cryptocurrencies at time (Binance Coin, Bitcoin Cash, Bitcoin, Cardano, Chainlink, Dogecoin, Ethereum Classic, Ethereum, Litecoin, Ripple, Stellar and Vechain), for differnet starting points but all ending at 18/10/2021. The contribution is to test whether or not GARCH models provide risk managers more accurate volatility forecasts for digital currencies. To answer this question a large empirical analysis is performed by comparing one-step-ahead volatility predictions from different specifications of GARCH models. GARCH specifications are up to four scedastic functions, and six error distributions, resulting in a total of twenty four estimated models for each cryptocurrency studied in this work. Finally, this work contributes by adding to the ongoing literature on digital coins the analysis of cryptocurrencies log-returns volatility dynamics focusing on different currencies and not solely on BitCoin. As stated by Corbet et al. (2018c), some cryptocurrencies has the dynamics that are relatively isolated to the others, which may offer diversification benefits from investors. Also, the variety of cryptocurrencies is still increasing, and the risk management of these assets has a growing place, demanding more accurate volatility prediction models.

After this introduction, this paper is organized as follows. Section 2 addresses the literature review on volatility modeling and forecasting for digital currencies. Empirical data and details on the methodology are described in Section 3. Section 4 provides the results and their corresponding discussion. Finally, Section 5 summarizes the main findings and suggests topics for future research.

## 2 Literature review

Literature on volatility within the cryptocurrencies market is still under development, but already finds strong contributions. The study of Phillip et al. (2018) sought to investigate the stylized facts of volatility in cryptocurrency markets. A sample of 224 cryptocurrencies were collected and the calculated returns were modeled in order to measure components like the generalized long memory (GLM) of Gray et al. (1989), leverage, and heavy tails. Through Taylor (2008) stochastic volatility model, the results found that: 1) as the market matures, cryptocurrencies tend to have similar characteristics of long memory persistence; 2) volatility and returns are negatively correlated when one day ahead; 3) there is a stochastic volatility process; 4) all cryptocurrencies under analysis present clustering volatility; 5) cryptocurrencies have heavy tail characteristics.

Other stylized facts were also found by Ma and Tanizaki (2019). Authors investigated the existence of the day-of-the-week effect on the return and volatility of Bitcoin, using data from January 2013 to December 2018, and stochastic volatility models. Results indicated the evidence of the day-of-the-week effect on Monday, which suggests a significantly higher mean return than other weekdays. A higher mean return was also associated with a higher volatility. The analysis suggested that the day-of-the-week effect on return equation varies with the sample period, while higher and more significant volatilities are observed on Monday and Thursday. Furthermore, the day-of-the-week effects on both returns and volatility are not justified by stock market returns (measured by exchange indexes) nor foreign exchange market returns(Ma and Tanizaki, 2019). In addition, no asymmetric effects were found in Bitcoin daily returns.

Kinateder and Papavassiliou (2019) studied calendar anomalies in daily Bitcoin returns and volatility, focusing on the Halloween, the day-of-the-week (DOW), and the month-of-the-year (MOY) calendar effects. Asymmetric GJR-GARCH (Glosten-Jagannathan-Runkle GARCH) model, from Glosten et al. (1993), with additional external regressors was applied. For the period from March 2013 to September 2019, the authors found no evidence of a Halloween effect on Bitcoin returns and volatility. Furthermore, there is no evidence of a classical DOW effect on returns, and Wednesday is the only DOW that exhibits anomalous behavior. On the other hand, a lower volatility is observed between Friday and Sunday, while it increases at the beginning of the week. In addition, the MOY effect is found in September, showing less volatility. The results validate the view that Bitcoin returns are mostly weak-form efficient with respect to calendar anomalies (Kinateder and Papavassiliou, 2019).

Volatility and market sentiment in cryptocurrency markets was evaluated by the literature. López-Cabarcos et al. (2019) investigated the dynamics of Bitcoin volatility and how sentiment on social networks (measured by Stocktwits), the stock market (measured by S&P500 index), and the volatility of the stock market (measured by the VIX index) influence Bitcoin volatility. Between January 2016 and September 2019, Bitcoin volatility was estimated according to a GARCH model with external regressors to test the influence of investor sentiment, the S&P500 and the VIX. The results indicated that investors should consider the stock market sentiment on social media when outlining their investment strategies. Furthermore, Bitcoin volatility behaves differently over time. Finally, according to López-Cabarcos et al. (2019), in times of high volatility in the stock market, Bitcoin can be used as a safe haven and, during periods of low volatility in the stock market, it could turn back for speculative purposes.

Understanding the dynamics of volatility is also important in measuring Value-At-Risk (VaR), a popular market risk measure. Liu et al. (2020) aimed to identify whether the VaR of cryptocurrencies could be predicted with EWMA (Exponential Weighted Moving Average) models, focusing in methods related to the Generalized Autoregressive Scoring (GAS) time series framework, proposed by Gerlach et al. (2013). For Bitcoin, Litecoin and Ethereum, EWMA VaR forecasts were evaluated in terms of the unconditional coverage test of Kupiec (1995), the conditional coverage test of Christoffersen (1998), the dynamic quantile test of Engle and Manganelli (2004), and the Model Confidence Set procedure of Hansen et al. (2011). The GAS specification consistently performed best for most of the VaR levels evaluated in the paper. Authors also concluded that in the Laplace related specifications, time-varying skewness asymmetric volatility responses may be preferred over the responses driven by the time-varying symmetric tails parameter.

Concerning GARCH-family models, Katsiampa (2017) investigated which conditional heteroskedasticity model could best describe Bitcoin volatility returns for the period from July 2010 to October 2016. The optimal model found was the standard Autoregressive GARCH (AR-GARCH), which demonstrates the importance of having a short-run and a long-run component to model the Bitcoin conditional variance.

In multivariate approach, Katsiampa (2019) evaluated volatility dynamics of Bitcoin, Ether, Ripple, Litecoin and Stellar Lumen price returns using an asymmetric multivariate GARCH model (Diagonal BEKK). The approach examined the conditional volatility dynamics and the correlation among the cryptocurrencies (i.e. their interdependence), and allowed asymmetric responses of negative and positive shocks to cryptocurrencie's conditional volatility and covariances. For the period from August 2015 to February 2018, the author verified that the conditional volatility of Bitcoin, Ether, Ripple and Litecoin showed asymmetric effects for good and bad news, as well as conditional covariances were found to be significantly affected by cross products of previous error terms and previous covariance terms. Hence, they concluded that significant volatility co-movements between cryptocurrencies do exist, empirically supporting the interdependence within the cryptocurrency market, which presented different behaviors over time. Further, Katsiampa (2019) noticed that spikes found in conditional variances, covariances and correlations, support the idea that cryptocurrencies volatilities are susceptible to news related to the digital currency markets.

Fakhfekh and Jeribi (2020) evaluated GARCH-family models for the volatility modeling of the sixteen largest cryptocurrencies at the time, under different error distributions (normal, student, generalized-error and double-exponential). Using data for the period from August 2017 to December 2018, the results showed that the Threshold GARCH (TGARCH) specification with double exponential distribution was the best volatility approach for most of the digital coins. Cerqueti et al. (2020) also evaluated the volatility prediction under non-Normal distributions for Bitcoin, Ethereum and Litecoin returns. From March 2014 to March 2019, the analysis concluded that the skewed specifications of the GARCH model represents the most effective way to predict the volatility of Bitcoin, Litecoin and Ethereum, with the predominance of the generalized error distribution in the cases of Bitcoin and Litecoin.

Similarly, Köchling et al. (2020) applied several GARCH models for Bitcoin volatility modeling and forecasting. Between November 2015 and October 2018, eleven variations of the GARCH model for the conditional variance and two conditional distributions (Normal and Student t), up to a total of

172 forecasting models were estimated. Volatility forecasts were calculated one-step-ahead at a rolling-window scheme. The results demonstrated that the Normal distribution provided a best performance..

Another way of approaching GARCH models has been the modeling together with machine learning models. Aras (2021) evaluated the effect of different model orders of the GARCH process on the volatility forecasts of Bitcoin obtained by four machine learning models: Artificial Neural Networks (ANN), Support Vector Machines (SVM), Random forest (RF) and K-nearest Neighbors (KNN). For the period between July 2013 and August 2020, the respective models were calibrated and volatility forecasts calculated every 25 days, on a rolling-window scheme. In turn, the stacking ensemble structure was proposed using the above models together with a standard GARCH model. The structure in question used feature selection techniques such random forest selector and principal component analysis to reduce the dimension of the predictors before meta-learning. The results showed that the proposed stacking ensemble has superior forecasting capacity when compared to hybrid GARCH models.

Peng et al. (2018) estimated the volatility of Bitcoin, Ethereum, Dash, and traditional foreign exchange rates, combining the traditional GARCH model with a machine learning approach, the Support Vector Regression (SVR), in estimating the mean and the volatility equations. The method was compared against other specifications of the GARCH family under Normal, Student's t and Skewed Student's t distributions. For January 2016 to July 2017, the SVR-GARCH model was applied to low (daily) and high (hour) frequency data. The results showed that the SVR-GARCH model is superior compared to the others for all distributions and all frequencies, according to the accuracy error metrics, and also considering the statistical Diebold-Mariano and Model Confident Set tests.

The literature in general advocates the suitability of GARCH-family models in modeling and forecasting the volatility dynamics of cryptocurrencies.

However, most of the works whether consider only Bitcoin or a few digital coins. Therefore, this paper investigates the volatility dynamics through GARCH-Family models for twelve leading cryptocurrencies for different starting dates, all ending at 18/10/2021, under different scedastic functions and error distributions, comprising a total of twenty four for each digital coin.

# 3 Methodology

This work evaluates the volatility modeling and forecasting of cryptocurrencies asset returns using GARCH-family models. A large empirical analysis is provided by considering different scedastic functions, error distributions and models structure (parametrization). All models are detailed in this section.

The GARCH-family models consider four different structures in terms of scedastic funcion: GARCH, Glosten-Jagannathan-Runkle GARCH (GJR-GARCH), Exponencial GARCH (EGARCH) and Component GARCH (CS-GARCH). In addition, for the errors parameters, the following distributions were considered: normal, skewed normal, Student's t, skewed Student's t, generalized error distribution and skewed generalized error distribution. The models are described below.

## 3.1 GARCH-family models

The Generalized Autoregressive Conditional Heteroskedacity (GARCH) family models consider the time series variance modeling as a conditional process over time.

Let $\{\epsilon_t\}$ be a stochastic process with zero mean and serially uncorrelated, we can model its variance according to the following structure:

$$\epsilon_t = \nu_t \sigma_t \tag{1}$$

where $\sigma_t^2$ represents the conditional variance process and $\nu_t$ a white-noise.

The functional form that describes conditional variance differentiates GARCH structures, while the error process $\nu_t$ can assume one of the following distributions: normal distribution (norm), skewed normal distribution (snorm), Student's t distribution (std), skewed Student's t distribution (sstd), generalized error distribution (ged), or skewed generalized error distribution (sged).

A GARCH($p, q$) model (Bollerslev, 1986) can be described:

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{q} \alpha_j \epsilon_{t-j}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \tag{2}$$

where $p$ is the number of lags of the conditional variance, $q$ is the number of lags from past shocks, and $\alpha_0, \alpha_j, \beta_i$ the corresponding parameters, with $i = 1, \ldots, p$ and $j = 1, \ldots, q$, such that $\alpha_0 > 0$, $\alpha_i \geq 0$, and $\beta_i \geq 0$.

A characteristic of the GARCH model is that it does not consider the asymmetry effect on volatility, that is, there is no distinction between the effects of negative and positive returns on the conditional variance. To take this issue into account, the GJR-GARCH model, developed by Glosten et al. (1993). The conditional variance of the GRJ-GARCH model($p, q$) is defined as:

$$\sigma_t^2 = \sum_{j=1}^{q} (\alpha_j \epsilon_{t-j}^2 + \gamma_j I_{t-j} \epsilon_{t-j}^2) + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \tag{3}$$

$I_{t-j}$ is a binary variable of the form:

$$I_{t-j} = \begin{cases} 1, & \text{if } \epsilon_{t-j} < 0, \quad \text{for } j = 1, 2, ..., q \\ 0, & \text{if } \epsilon_{t-j} \geq 0, \quad \text{for } j = 1, 2, ..., q \end{cases}$$

The parameter $\gamma$ captures the asymmetry effect, if significant, there is a distinct impact on the conditional variance process of positive and negative returns. In the GJR-GARCH model, the constraints on the parameters are such that $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_i \geq 0$, $\alpha_i + \gamma_i \geq 0$.

Similar to the GJR-GARCH model, the EGARCH model, developed by Nelson (1991), it also takes into account asymmetric effects, however, its structure avoids the need for limitations on the positivity of the parameters. An EGARCH$(p, q)$ model can be represented as follows:

$$ln(\sigma_t^2) = \sum_{j=1}^{q}(\alpha_j \nu_{t-j} + \gamma_j(|\nu_{t-j}| - E|\nu_{t-j}|)) + \sum_{i=1}^{p} \beta_i ln(\sigma_{t-i}^2) \tag{4}$$

Again, $\gamma$ is the parameter that captures the asymmetry effect in the process generating the dynamics of conditional variance.

Finally, the Component GARCH (CS-GARCH), from Engle et al. (1999), describes conditional volatility as a permanent and a transitory component. A CS-GARCH$(p, q)$ model can be represented as:

$$\sigma_t^2 = q_t + \sum_{j=1}^{q} \alpha_j(\epsilon_{t-j}^2 - q_{t-j}) + \sum_{i=1}^{p} \beta_i(\sigma_{t-i}^2 - q_{t-i}) \tag{5}$$

$$q_t = \omega + pq_{t-1} + \phi(\epsilon_{t-1}^2 - \sigma_{t-1}^2)$$

where $q_t$ epresents the permanent component and $\sigma_{t-j}^2 - q_{t-j}$ the transitory component, such as difference between the conditional variance and its trend.

For all models, the parametrization, i.e. definition of the orders $p$ and $q$ will be according to the Bayesian information criterion (BIC).

## 3.2 Volatility performance metrics

The estimated volatility models will be used to generate forecasts one step ahead. To assess the quality of the predictions, different loss functions will be considered. Loss functions will be computed for out-of-sample data. Be $T$ the number of observations in the sample, $\hat{\sigma}_t^2$ the variance predicted by the GARCH models, and $\sigma_t^2$ the actual observed variance. Note that volatility is an unobserved measure. In this sense, $\sigma_t^2 = r_t^2$, where $r_t^2$ is squared at the instant $t$. As a loss function are considered the Mean Square Error (MSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), and QLIKE, from Patton (2011). AAdjustment measures are calculated as:

$$MSE = \frac{1}{T}\sum_{t=1}^{T}(\sigma_t^2 - \hat{\sigma}_t^2)^2 \tag{6}$$

$$MAE = \frac{1}{T}\sum_{t=1}^{T}|\sigma_t^2 - \hat{\sigma}_t^2| \tag{7}$$

$$MAPE = \frac{1}{T}\sum_{t=1}^{T}|\frac{\sigma_t^2 - \hat{\sigma}_t^2}{\sigma_t^2}| \tag{8}$$

$$QLIKE = \frac{1}{T}\sum_{t=1}^{T}(ln(\hat{\sigma}_t^2) + \frac{\sigma_t^2}{\hat{\sigma}_t^2}) \tag{9}$$

For the considered accuracy measures, the lowest values are associated with models with better predictive performance.

# 4 Empirical experiments

## 4.1 Data

According to coinmarketcap.com, the global capitalization of the cryptocurrency market already exceeds \$1.90 trillion, with the market consisting of over 16,000 cryptocurrencies in circulation[7]. This paper selected the twelve cryptocurrencies with the highest market capitalization (for September 2021) that have more than 3 years of historical data avaiable and are not stablecoins. The sample comprised the following digital coins: Binance Coin, Bitcoin Cash, Bitcoin, Cardano, Chainlink, Dogecoin, Ethereum Classic, Ethereum, Litecoin, Ripple, Stellar, and Vechain.

[7]Source: https://www.explodingtopics.com/blog/number-of-cryptocurrencies. Access on 11, March, 2022.

Data was collected through the coingecko.com API, as the platform provides an average weighting of the traded value of each cryptocurrency in the market[8], a proxy that best reflects the dynamics of market prices. Data was collected for multiple starts date, due to availability of historical prices, but all ending on September 18, 2021. Closing prices data were divided into in-sample and out-of-sample sets. In-sample data comprises the initial 70% of total observations and is used for models parametrization. Finally, the out-of-sample set, with the last 30% of total observations, is used for models testing. Data details are provided in Table 1.

Table 1: Cryptocurrency data description.

| Crypto | In-Sample Starting Date | # Obs. | Out-of-Sample Starting Date | # Obs. | Total Obs. |
|---|---|---|---|---|---|
| Binance Coin | 17/09/2017 | 1044 | 27/07/2020 | 448 | 1492 |
| Bitcoin Cash | 01/08/2017 | 1077 | 13/07/2020 | 462 | 1539 |
| Bitcoin | 02/07/2013 | 2121 | 23/04/2019 | 909 | 3030 |
| Cardano | 17/10/2017 | 1023 | 05/08/2020 | 439 | 1462 |
| Chainlink | 08/11/2017 | 1008 | 12/08/2020 | 432 | 1440 |
| Dogecoin | 17/12/2013 | 2003 | 12/06/2019 | 859 | 2862 |
| Ethereum Classic | 23/07/2016 | 1339 | 23/03/2020 | 574 | 1913 |
| Ethereum | 07/08/2015 | 1585 | 09/12/2019 | 679 | 2264 |
| Litecoin | 02/07/2013 | 2121 | 23/04/2019 | 909 | 3030 |
| Ripple | 08/08/2013 | 2095 | 04/05/2019 | 898 | 2993 |
| Stellar | 11/08/2014 | 1837 | 22/08/2019 | 788 | 2625 |
| Vechain | 26/07/2018 | 826 | 29/10/2020 | 354 | 1180 |

Closing prices were used to compute the respective log returns as $r_t = ln(P_t) - ln(P_{t-1})$, where $P_t$ is the closing price at day $t$. The return data descriptive statistics are summarized in Table 2. Additionally, Figures 1-3 shows the plots of the corresponding returns.

[8]Source: www.coingecko.com/en/methodology. Access on 10, March, 2022.

Table 2: Summary statistics of the return data.

| Crypto | Mean | Std. Dev. | Min | Max | Median | Skewness | Kurtosis | # Obs. |
|---|---|---|---|---|---|---|---|---|
| Binance Coin | 0,006 | 0,124 | -1,002 | 3,270 | 0,001 | 11,718 | 328,674 | 1494 |
| Bitcoin Cash | 0,000 | 0,077 | -0,655 | 0,875 | -0,001 | 0,410 | 24,722 | 1541 |
| Bitcoin | 0,002 | 0,042 | -0,434 | 0,287 | 0,002 | -0,556 | 11,726 | 3032 |
| Cardano | 0,003 | 0,071 | -0,524 | 0,872 | 0,002 | 1,826 | 26,460 | 1464 |
| Chainlink | 0,003 | 0,076 | -0,661 | 0,476 | 0,000 | -0,032 | 10,932 | 1442 |
| Dogecoin | 0,002 | 0,085 | -0,940 | 1,479 | -0,002 | 2,748 | 58,373 | 2864 |
| Ethereum Classic | 0,002 | 0,073 | -0,534 | 1,298 | 0,000 | 2,794 | 58,506 | 1915 |
| Ethereum | 0,003 | 0,065 | -0,755 | 0,440 | 0,001 | -1,249 | 21,615 | 2266 |
| Litecoin | 0,001 | 0,062 | -0,547 | 0,659 | -0,001 | 0,715 | 19,993 | 3032 |
| Ripple | 0,002 | 0,073 | -0,913 | 0,881 | -0,001 | 0,944 | 29,905 | 2995 |
| Stellar | 0,002 | 0,092 | -1,053 | 1,633 | -0,001 | 1,955 | 60,790 | 2627 |
| VeChain | 0,001 | 0,069 | -0,609 | 0,352 | 0,001 | -0,502 | 11,113 | 1182 |

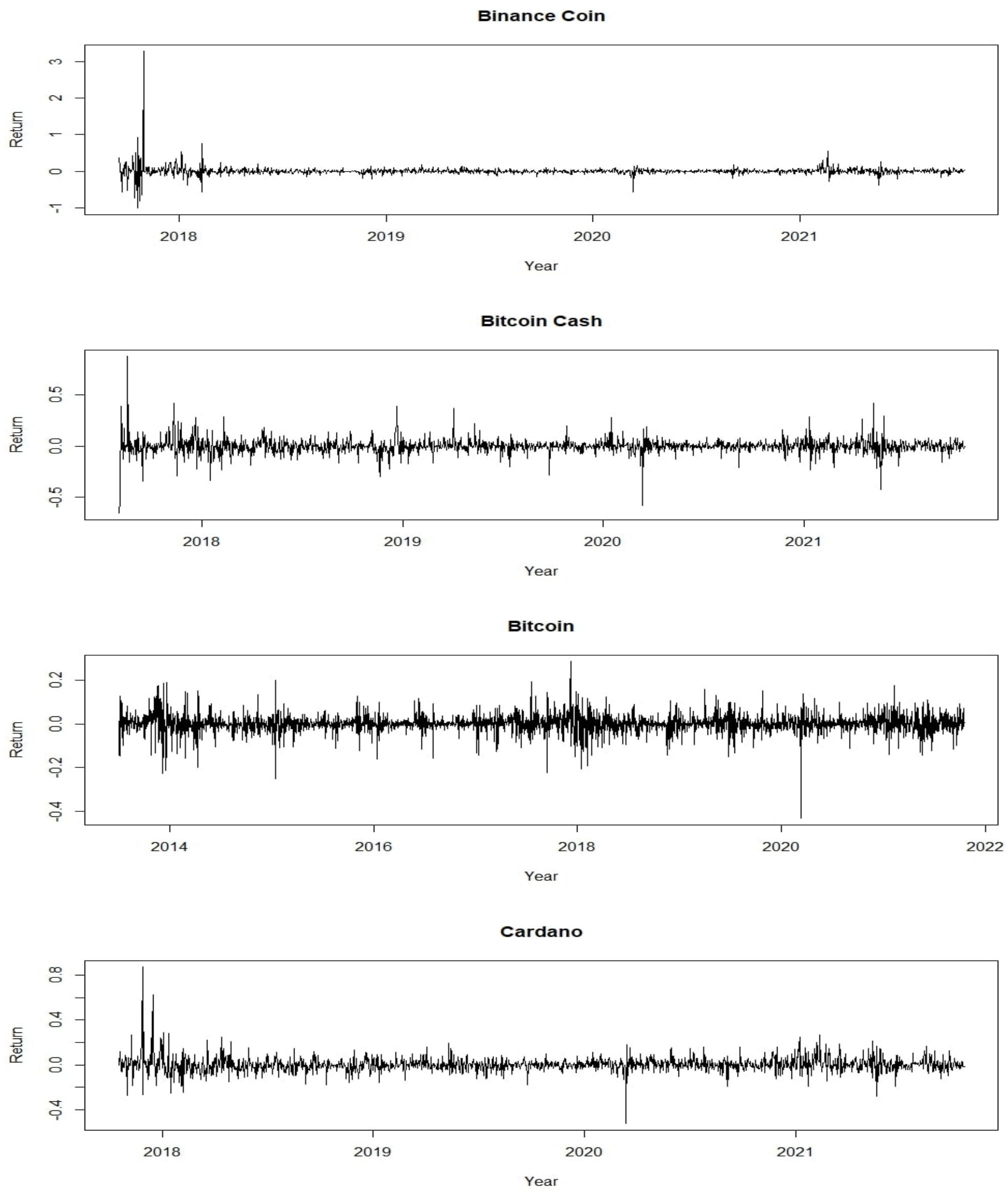


Figure 1: Temporal evolution of daily returns of Binance Coin, Bitcoin Cash, Bitcoin and Cardano.

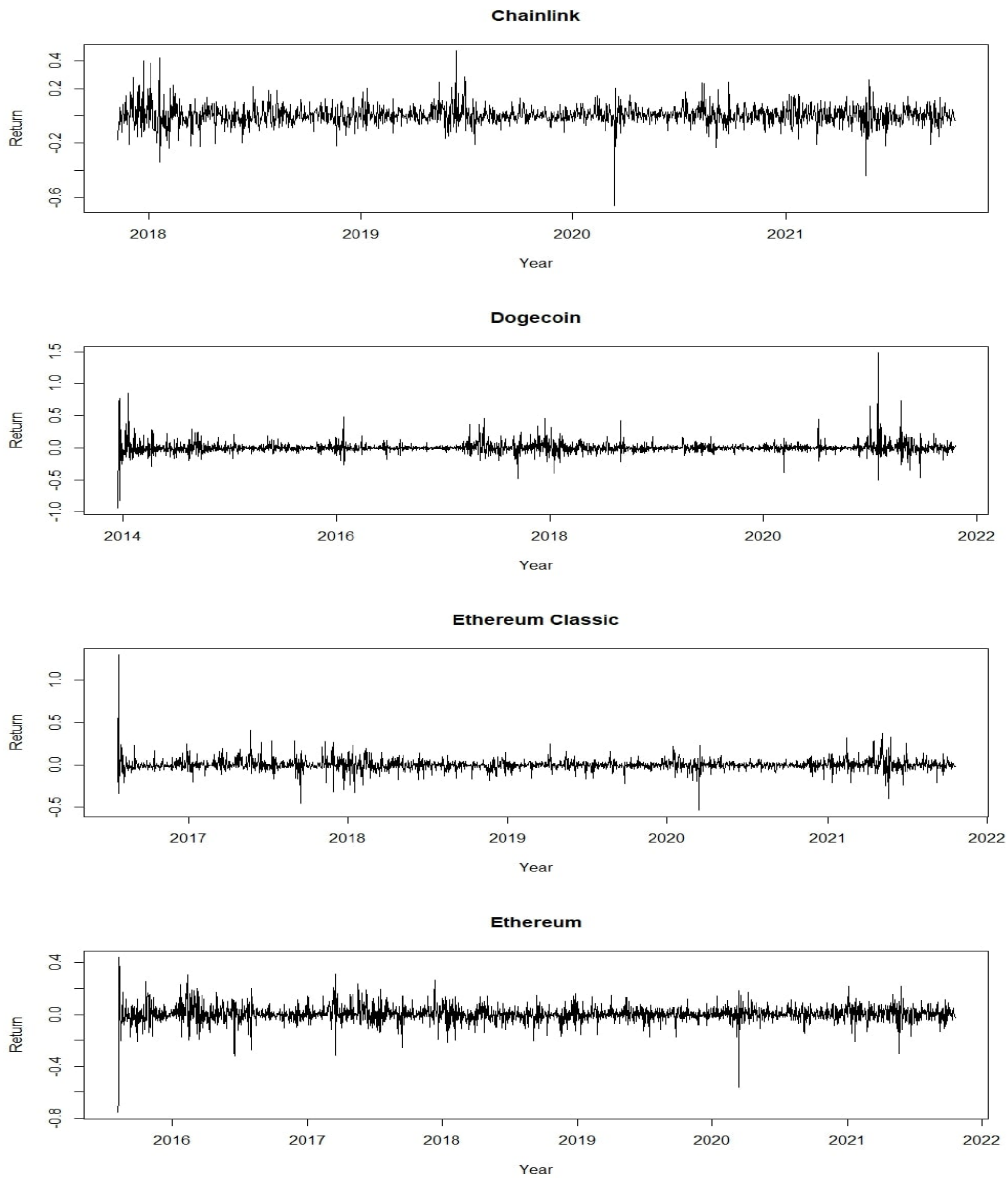


Figure 2: Temporal evolution of daily returns of Chainlink, Dogecoin, Ethereum Classic and Ethereum.

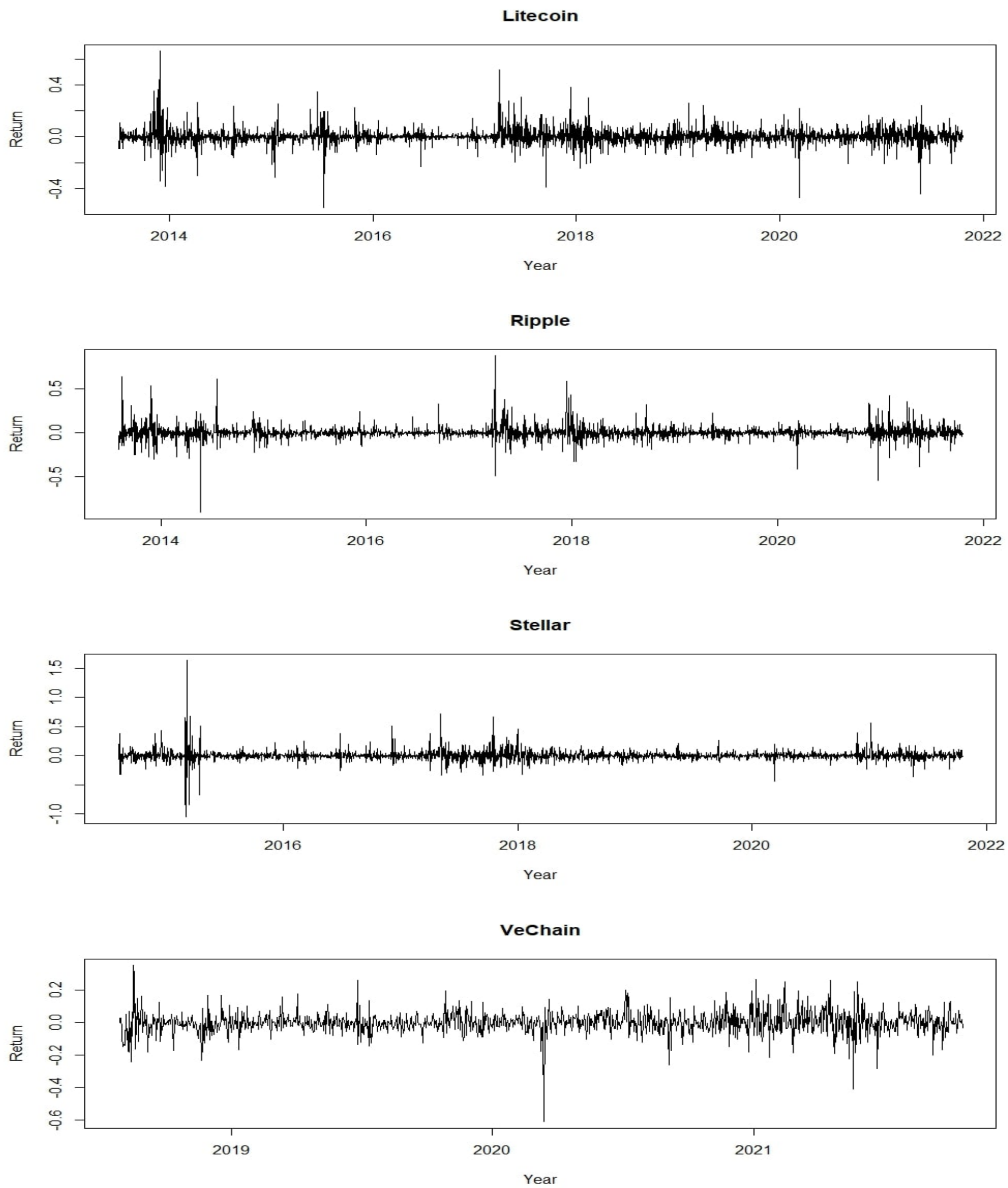


Figure 3: Temporal evolution of daily returns of Litecoin, Ripple, Stellar and Vechain.

From Table 2, one must notice the presence of skewness (negative and positive) and high values of kurtosis implying moments of extreme returns (positive or negative) for all cryptocurrencies evaluated in this work. Further, the analysis of the temporal evolution of price returns evidences the time-varying dynamic of volatility, with the presence of common stylized facts such as volatility clustering (see Figures 1-3).

## 4.2 Volatility modeling: in-sample analysis

Using in-sample data, models were evaluated in order to select the best GARCH structure for each digital coin based on the Bayesian information criteria (BIC). These structures were used in the subsequent out-of-sample analysis, in which the volatility forecasts of the one-step-ahead is conducted. Hence, for each series of cryptocurrencies, the best model among GARCH, GJR-GARCH, EGARCH and CSGARCH, was selected according to the corresponding BIC value. The lower the BIC the best the model in terms of parsimony (model fits the data well and also provide a few number of parameters). The results of models selection are summarized in Table 3. In this table, the best structure is presented for each scedastic function, associated with the best error distribution. Among the scedastic functions, the best model is highlighted with asterisk ($*$). Notice that, from Table 3, for Bitcoin Cah, Dogecoin and Ethereum, only one scedastic function provided suitable returns, in terms of the corresponding residual analysis. The remaing scedastic functions were note able to produce robust results in terms of capturing the time-varying volatility of returns (i.e. residuals showed autocorrelation). It is worth to mention that, for some cryptocurrencies, returns mean must be filttered by an Autoregressive Moving Average (ARMA) model to account for autocorrelation. The need of this correction was evaluated through the Ljung-Box test for autocorrelation.

Table 3: Models selection.

| **Structures** | **BIC** |
|---|---|
| Binance Coin - ARMA(2,1) - sGARCH(1,1) - ged | -2,9064 |
| Binance Coin - ARMA(2,1) - gjrGARCH(1,1) - ged | -2,9011 |
| Binance Coin - ARMA(2,1) - eGARCH(1,1) - ged | -2,9122 |
| *Binance Coin - ARMA(2,1) - csGARCH(1,1) - std | -2,9149 |
| *Bitcoin Cash - ARMA(5,2) - csGARCH(1,1) - std | -2,8246 |
| Bitcoin - ARMA(5,1) - sGARCH(1,1) - ged | -4,0656 |
| Bitcoin - ARMA(5,1) - gjrGARCH(1,1) - ged | -4,0623 |
| *Bitcoin - ARMA(5,1) - eGARCH(1,1) - ged | -4,0663 |
| Bitcoin - ARMA(5,1) - csGARCH(1,1) - ged | -4,0652 |
| Cardano - ARMA(3,0) - sGARCH(1,1) - std | -2,8864 |
| Cardano - ARMA(3,0) - gjrGARCH(1,1) - std | -2,8811 |
| Cardano - ARMA(3,0) - eGARCH(1,1) - std | -2,8800 |
| *Cardano - ARMA(3,0) - csGARCH(1,1) - std | -2,8901 |
| *Chainlink - sGARCH(1,1) - sstd | -2,6097 |
| Chainlink - gjrGARCH(1,1) - sstd | -2,6033 |
| Chainlink - eGARCH(1,1) - sstd | -2,6054 |
| Chainlink - csGARCH(1,1) - sstd | -2,5997 |
| *Dogecoin - ARMA(5,2) - csGARCH(2,1) - sstd | -3,2053 |
| Ethereum Classic - ARMA(5,3) - sGARCH(1,1) - std | -2,9608 |
| *Ethereum Classic - ARMA(5,3) - gjrGARCH(1,1) - std | -2,9647 |
| Ethereum Classic - ARMA(5,3) - eGARCH(1,1) - std | -2,9629 |
| Ethereum Classic - ARMA(5,3) - csGARCH(1,1) - sged | -2,9597 |
| *Ethereum - ARMA(3,1) - csGARCH(1,1) - ged | -3,0311 |
| Litecoin - ARMA(2,4) - sGARCH(1,1) - std | -3,5101 |
| Litecoin - ARMA(2,4) - gjrGARCH(1,1) - std | -3,5066 |
| *Litecoin - ARMA(2,4) - eGARCH(1,1) - sstd | -3,5288 |
| Litecoin - ARMA(2,4) - csGARCH(1,1) - sstd | -3,5129 |
| Ripple - ARMA(2,3) - sGARCH(1,2) - std | -3,2514 |
| Ripple - ARMA(2,3) - gjrGARCH(2,1) - std | -3,2484 |
| Ripple - ARMA(2,3) - eGARCH(1,1) - sstd | -3,2707 |
| *Ripple - ARMA(2,3) - csGARCH(2,2) - std | -3,2787 |
| Stellar - ARMA(5,1) - sGARCH(1,1) - sstd | -2,7408 |
| Stellar - ARMA(5,1) - gjrGARCH(1,1) - std | -2,7346 |
| *Stellar - ARMA(5,1) - eGARCH(1,1) - sstd | -2,7464 |
| Stellar - ARMA(5,1) - csGARCH(1,1) - sstd | -2,7406 |
| *Vechain - sGARCH(1,1) - sstd | -3,0362 |
| Vechain - gjrGARCH(1,1) - sstd | -3,0282 |
| Vechain - eGARCH(1,1) - sstd | -3,0257 |
| Vechain - csGARCH(1,1) - sstd | -3,0191 |

From Table 3, there is consistency in the distribution of errors among the selected models. Although there are exceptions, each series tends to converge to a certain type of distribution, regardless of the GARCH model. Furthermore, only the skewed t-student, GED - and t-student distributions occur, with the skewed t-student distribution predominating. This means that the models tend to have asymmetric distributions, supporting the idea that negative returns affect the modeling of the series more than positive returns.

In addition, most of the models with the best fit had GARCH (1,1) parameters (see Table 3). As shown in the literature, simpler models that explain the response variable well, i.e. parsimonious models are more consistent over time compared to more complex models that account for larger lags (a higher number of parameters). Figures 4-6 show the temporal evolution of the corresponding estimated volatilities (obtained from the best model in Table 3) for each cryptocurrency with the associated squared returns series. It is important to notice that GARCH-family models are able to model appropriately the time-varying variance of digital coins, especially in periods of high price variations (higer returns).

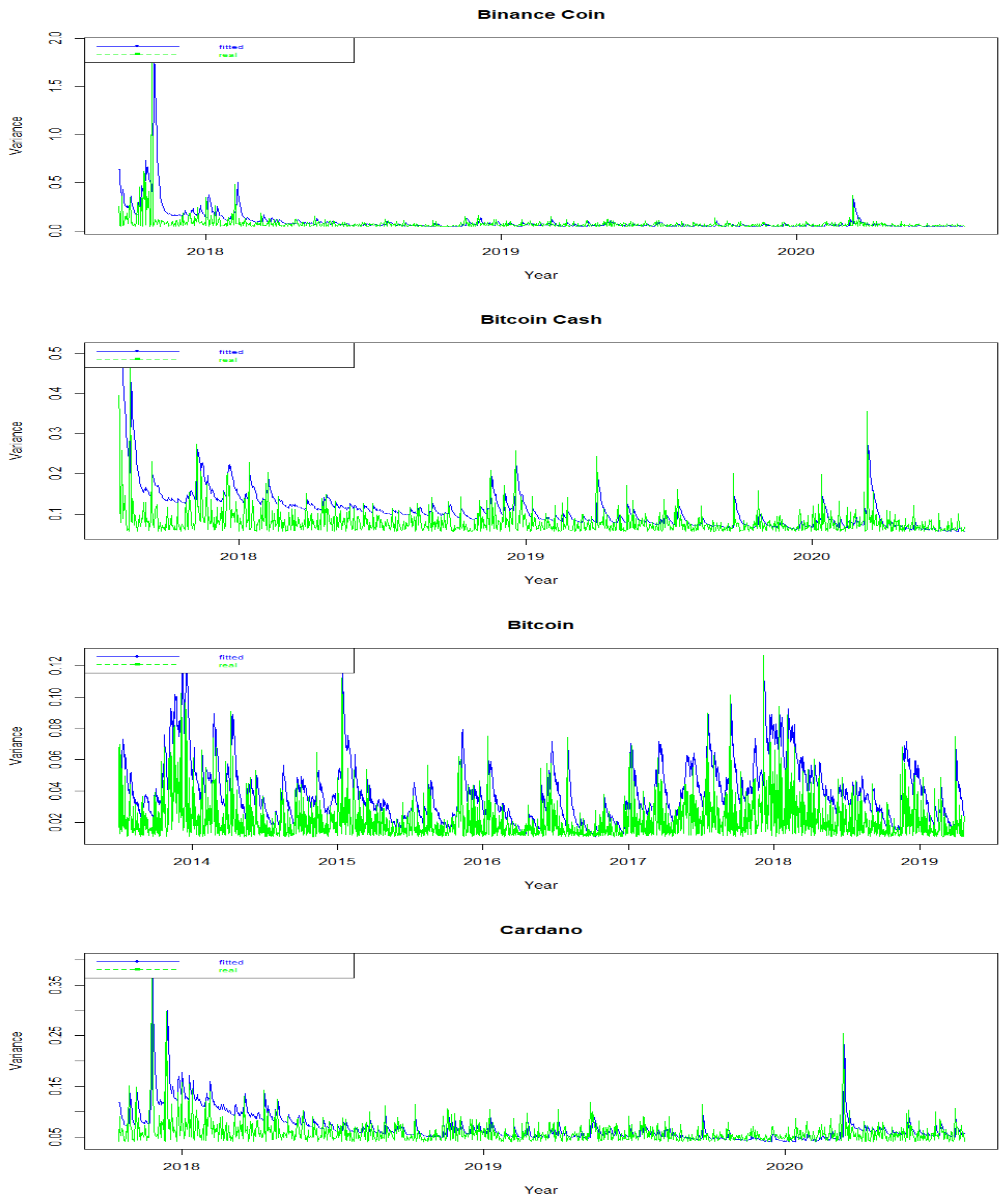


Figure 4: Temporal evolution of daily squared returns and the corresponding estimated volatility of Binance Coin, Bitcoin Cash, Bitcoin and Cardano.

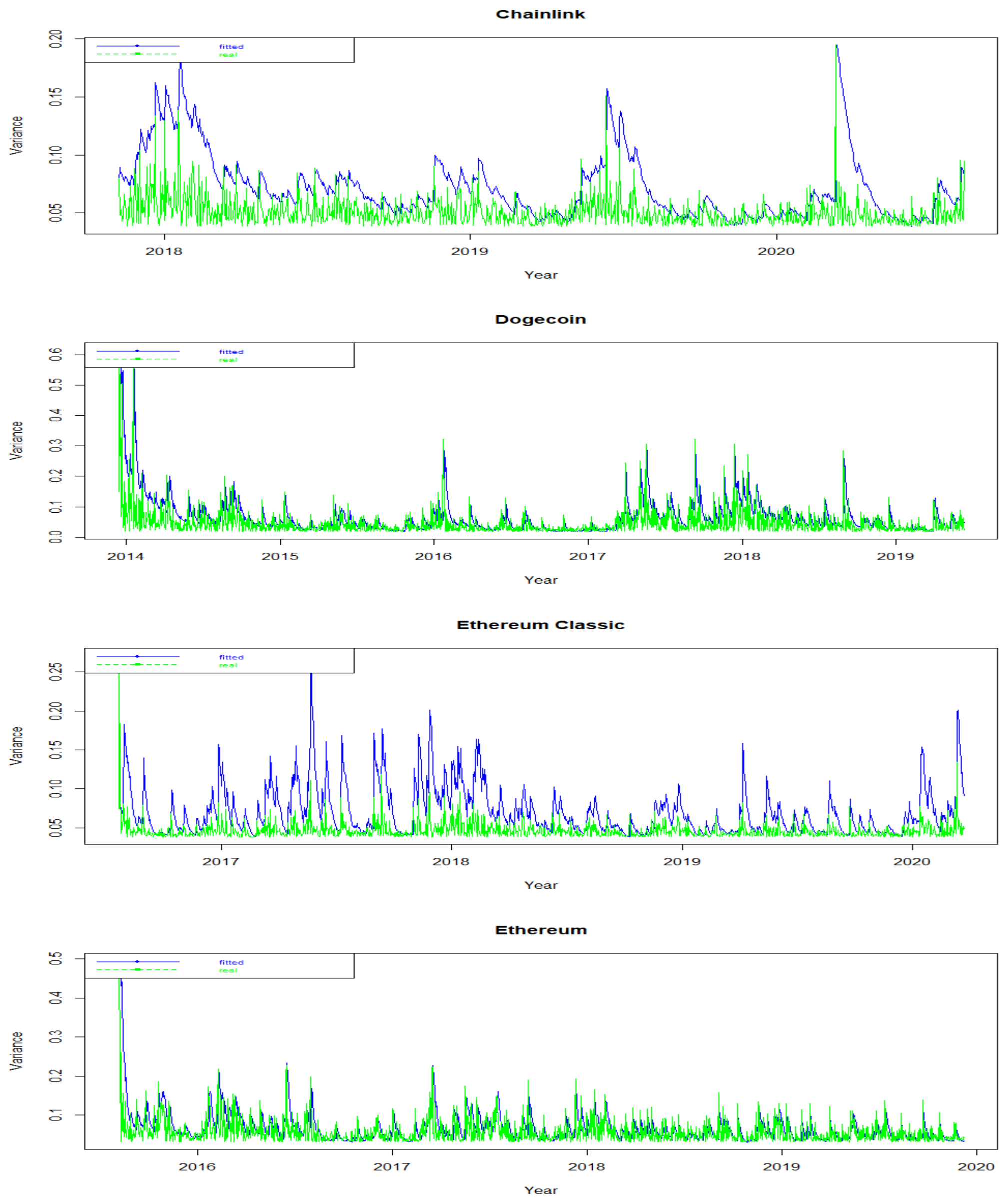


Figure 5: Temporal evolution of daily squared returns and the corresponding estimated volatility of Chainlink, Dogecoin, Ethereum Classic and Ethereum.

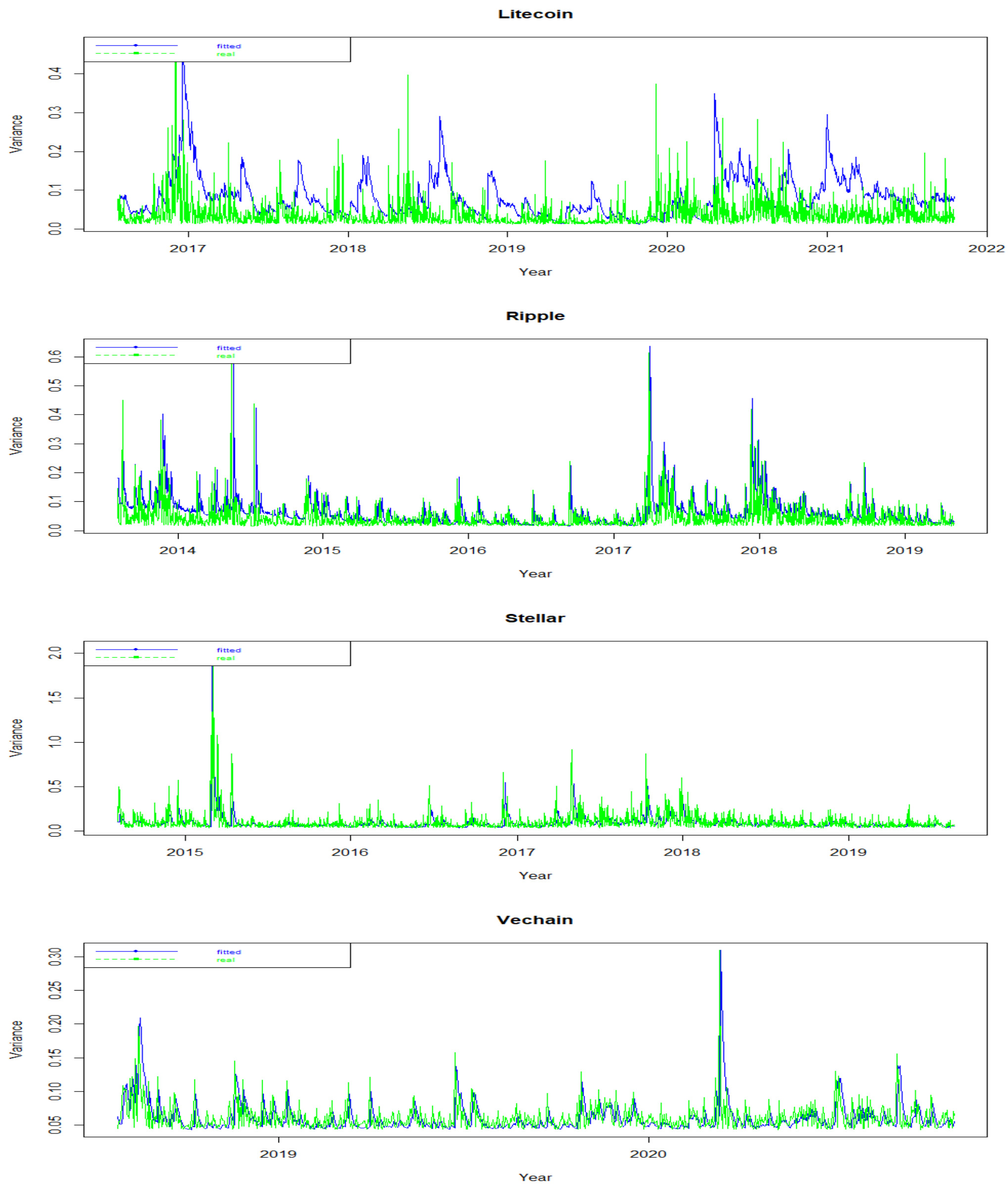


Figure 6: Temporal evolution of daily squared returns and the corresponding estimated volatility of Litecoin, Ripple, Stellar and Vechain.

## 4.3 Volatility forecasting: out-of-sample analysis

For each cryptocurrency series and GARCH scedastic function, models volatility forecasts for one-step-ahead were compared in ther of the functions MSE, MAE, MAPE, and QLike. The corresponding values of the error measures are provided in Table 4. Best results are highlighted in bold.

In general, results from Table 4 indicate the CSGARCH model as the most appropriate structure for volatility forecasting in the cryptocurrency market. This predominance may be related to the way CSGARCH models predict volatility. The CSGARCH, from Engle et al. (1999), divides variance into permanent and transitory components, allowing shocks to be transitory and distinguishing short-term from long-term volatility.

The GJRGARCH and EGARCH models (Glosten et al., 1993; Nelson, 1991) also allow volatility to be modeled in a particular context. In these cases, the conditional variance accounts for asymmetric effects, where positive and negative returns have a stylized impact on the model.

Figures 7-9 provides the volatility forecasts of the corresponding best GARCH models for all cryptocurrencies evaluated in this work for the out-of-sample sets. In most of the cases, GARCH approaches were able to properly forecast the one-step-ahead values of the conditional variance of digital coins returns, except for Cardano and Chainlink, where volatility is generally overestimated.

Table 4: Loss functions results for one-step-ahead forecasting of cryptocurrencies using GARCH-family models.

| **Model Structure** | **MSE** | **MAE** | **MAPE** | **QLIKE** |
|---|---|---|---|---|
| Binance Coin - ARMA(2,1) - sGARCH(1,1) - ged | 0,00032 | 0,00574 | **5971** | -2,18 |
| *Binance Coin - ARMA(2,1) - gjrGARCH(1,1) - ged | **0,00032** | **0,0056** | 6069 | -2,36 |
| Binance Coin - ARMA(2,1) - eGARCH(1,1) - ged | 0,00032 | 0,00561 | 8232 | **-1,99** |
| Binance Coin - ARMA(2,1) - csGARCH(1,1) - std | 0,00033 | 0,00659 | 8998 | -6,01 |
| *Bitcoin Cash - ARMA(5,2) - csGARCH(1,1) - std | 0,00022 | 0,00645 | 2706 | -5,65 |
| Bitcoin - ARMA(5,1) - sGARCH(1,1) - ged | 0,00005 | 0,0021 | 29173 | **10,72** |
| Bitcoin - ARMA(5,1) - gjrGARCH(1,1) - ged | 0,00005 | 0,00209 | **28167** | 12,2 |
| Bitcoin - ARMA(5,1) - eGARCH(1,1) - ged | 0,00005 | 0,00215 | 28342 | 12,4 |
| *Bitcoin - ARMA(5,1) - csGARCH(1,1) - ged | **0,00005** | **0,00204** | 31545 | 14,94 |
| *Cardano - ARMA(3,0) - sGARCH(1,1) - std | **0,00008** | 0,00488 | **7764519** | **-6,47** |
| Cardano - ARMA(3,0) - gjrGARCH(1,1) - std | 0,00008 | **0,00492** | 8678580 | -6,49 |
| Cardano - ARMA(3,0) - eGARCH(1,1) - std | 0,00008 | 0,00516 | 8956602 | -7,28 |
| Cardano - ARMA(3,0) - csGARCH(1,1) - std | 0,00008 | 0,00494 | 8381883 | -6,9 |
| Chainlink - sGARCH(1,1) - sstd | **0,00015** | 0,00575 | 67344 | -5,68 |
| *Chainlink - gjrGARCH(1,1) - sstd | 0,00015 | **0,00572** | 69099 | **-5,53** |
| Chainlink - eGARCH(1,1) - sstd | 0,00015 | 0,00584 | **66310** | -6,37 |
| Chainlink - csGARCH(1,1) - sstd | 0,00015 | 0,00593 | 70572 | -6,34 |
| *Dogecoin - ARMA(5,2) - csGARCH(2,1) - sstd | 0,00689 | 0,01165 | 5178 | 197,48 |
| Ethereum Classic - ARMA(5,3) - sGARCH(1,1) - std | 0,00019 | 0,0063 | 520 | -6,48 |
| Ethereum Classic - ARMA(5,3) - gjrGARCH(1,1) - std | 0,00021 | 0,00657 | 543 | -6,88 |
| Ethereum Classic - ARMA(5,3) - eGARCH(1,1) - std | 0,00022 | 0,00768 | 773 | -8,43 |
| *Ethereum Classic - ARMA(5,3) - csGARCH(1,1) - sged | **0,00018** | **0,00589** | **457** | **-6,36** |
| *Ethereum - ARMA(3,1) - csGARCH(1,1) - ged | 0,0002 | 0,00415 | 1952 | -0,72 |
| Litecoin - ARMA(2,4) - sGARCH(1,1) - std | 0,00013 | 0,00375 | Inf | -0,27 |
| Litecoin - ARMA(2,4) - gjrGARCH(1,1) - std | 0,00013 | 0,00374 | Inf | **-0,05** |
| Litecoin - ARMA(2,4) - eGARCH(1,1) - sstd | 0,00017 | 0,00707 | Inf | -8,29 |
| *Litecoin - ARMA(2,4) - csGARCH(1,1) - sstd | **0,00012** | **0,00365** | Inf | 0,66 |
| *Ripple - ARMA(2,3) - sGARCH(1,2) - std | **0,00031** | 0,00557 | 13124 | **4,67** |
| Ripple - ARMA(2,3) - gjrGARCH(2,1) - std | 0,00032 | 0,00564 | 16055 | 5,4 |
| Ripple - ARMA(2,3) - eGARCH(1,1) - sstd | 0,00041 | 0,00962 | 37740 | -6,94 |
| *Ripple - ARMA(2,3) - csGARCH(2,2) - std | 0,00032 | **0,00525** | **12713** | 18,25 |
| Stellar - ARMA(5,1) - sGARCH(1,1) - sstd | 0,0003 | 0,0062 | 4488 | -4,59 |
| Stellar - ARMA(5,1) - gjrGARCH(1,1) - std | 0,00030 | 0,00624 | 4454 | **-4,07** |
| Stellar - ARMA(5,1) - eGARCH(1,1) - sstd | **0,00028** | 0,00651 | 5222 | -5,26 |
| *Stellar - ARMA(5,1) - csGARCH(1,1) - sstd | 0,00029 | **0,00605** | **4265** | -4,11 |
| Vechain - sGARCH(1,1) - sstd | 0,0002 | 0,00701 | 29251 | 1,13 |
| Vechain - gjrGARCH(1,1) - sstd | 0,0002 | 0,00699 | **28691** | 1,05 |
| *Vechain - eGARCH(1,1) - sstd | **0,0002** | **0,00678** | 29538 | **-0,39** |
| Vechain - csGARCH(1,1) - sstd | 0,0002 | 0,00694 | 29453 | 1,22 |

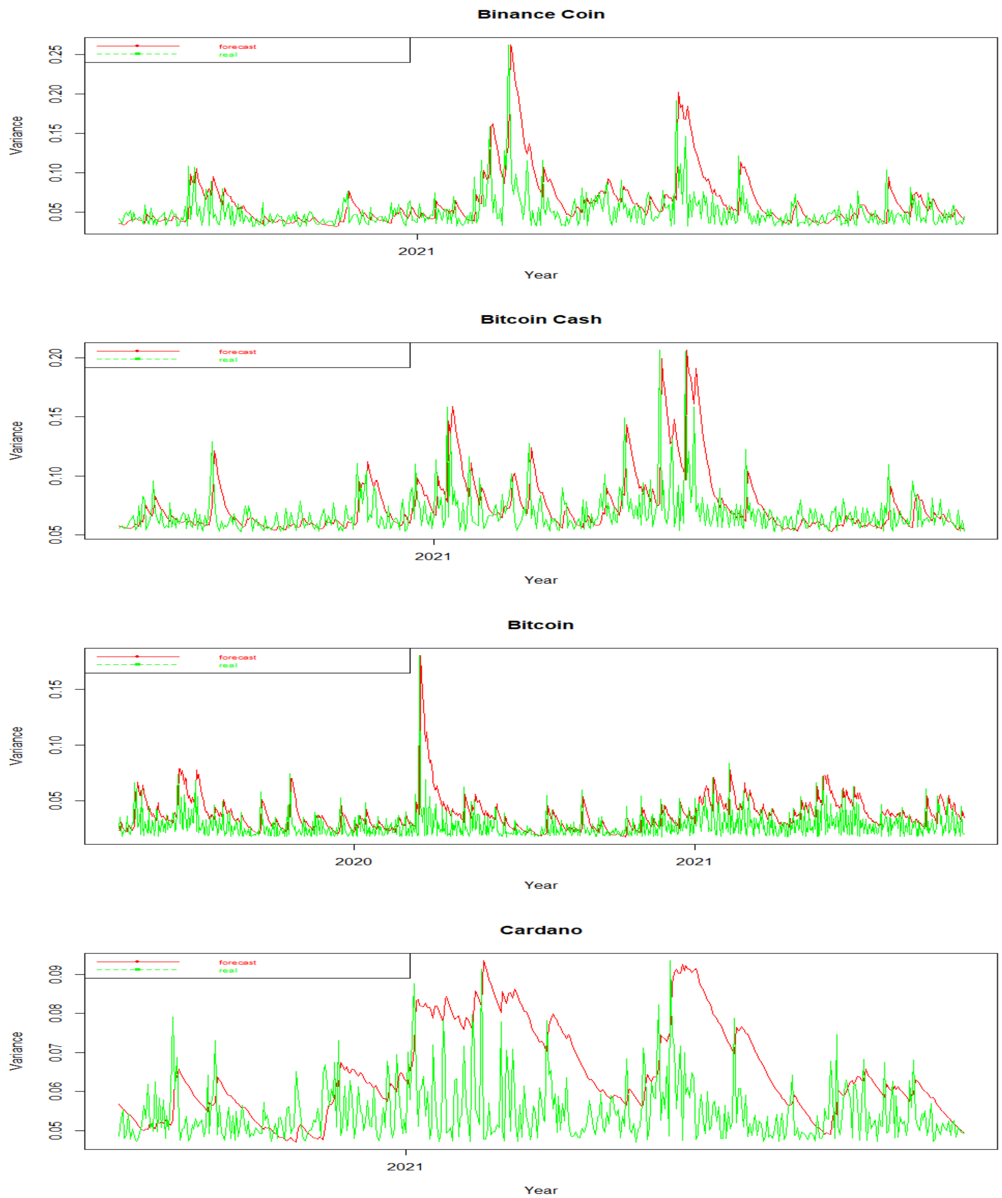


Figure 7: Temporal evolution of daily squared returns and the corresponding volatility forecasts for one-step-ahead of Binance Coin, Bitcoin Cash, Bitcoin and Cardano.


Figure 8: Temporal evolution of daily squared returns and the corresponding volatility forecasts for one-step-ahead of Chainlink, Dogecoin, Ethereum Classic and Ethereum.

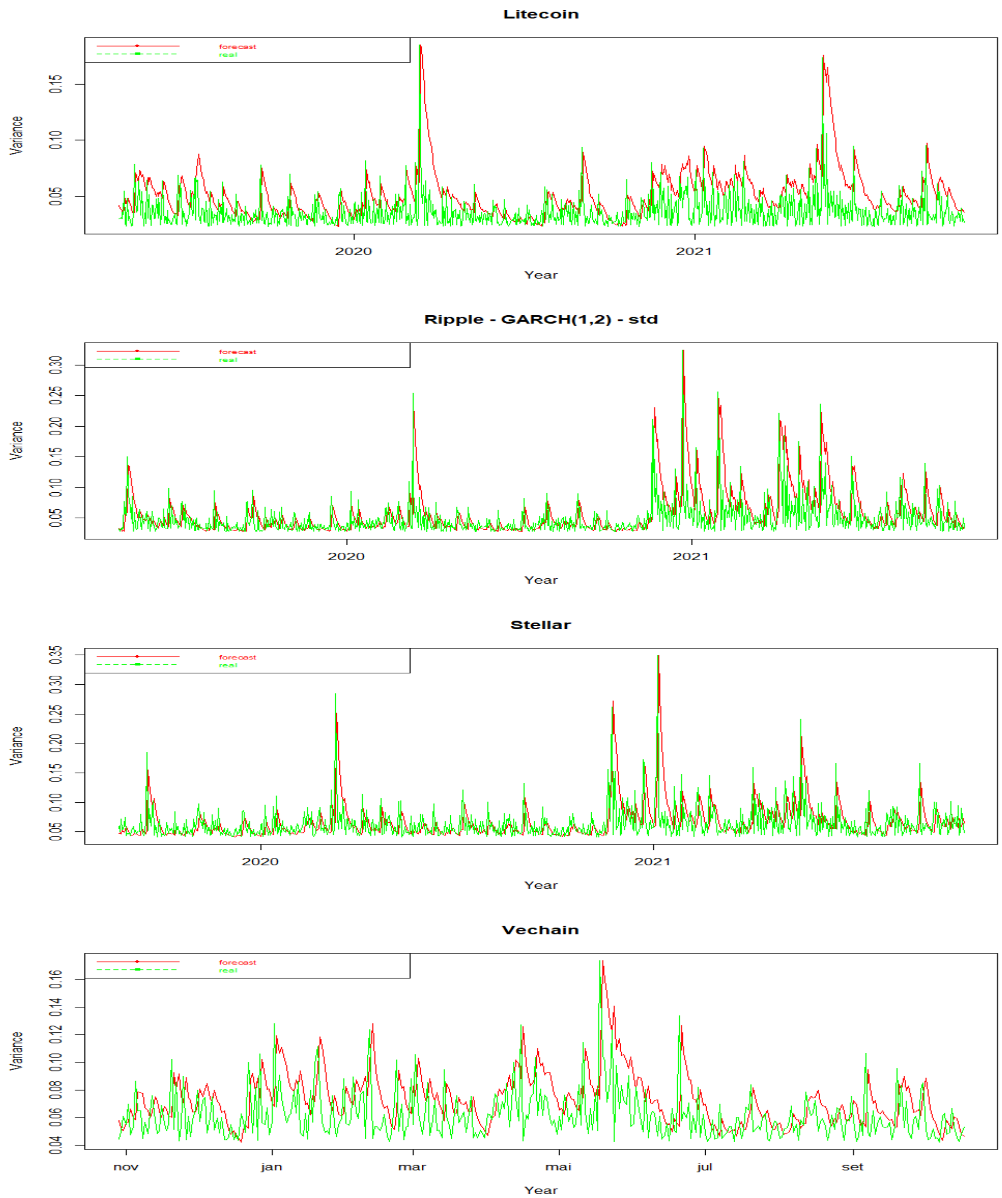


Figure 9: Temporal evolution of daily squared returns and the corresponding volatility forecasts for one-step-ahead of Litecoin, Ripple, Stellar and Vechain.

# 5 Conclusion

This paper studied the modeling and forecasting of volatility for the twelve largest cryptocurrencies in terms of market capitalization in September 2021 (Binance Coin, Bitcoin Cash, Bitcoin, Cardano, Chainlink, Dogecoin, Ethereum Classic, Ethereum, Litecoin, Ripple, Stellar, and Vechain). Considering four scedastic functions from GARCH-family models (GARCH, GJR-GARCH, EGARCH and CSGARCH) and five different error distributions (normal, skewed normal, Student's t, skewed Student's t, generalized error distribution and skewed generalized error distribution), models were evaluated in terms of forecasting performance using four loss functions (MSE, MAE, MAPE, and QLike). The forecasts were one-step-ahead.

Generally, with the exception of the cases of Bitcoin Cash, Dogecoin and Ethereum, for which only the CSGARCH models were appropriate, there was no consistency between in-sample and out-of-sample models, i.e. the best fitted model is not the one associated with the highest accuracy for the out-of-sample set. Moreover, the CSGARCH structure prevails as the best approach in in-sample modeling and in the analysis of out-of-sample forecasting accuracy. By splitting the conditional variance into a permanent and a transitory component, the CSGARCH model allows for transitory shocks in volatility modeling. It allows the short-term shock to not have a strong impact on long-term volatility. Given the strong volatility of the cryptocurrency market, CSGARCH is able to better capture this effect that occurs in these series. This component is consistent with the work of Katsiampa (2017) and Fakhfekh and Jeribi (2020), who found long-term effects in cryptocurrencies.

Similarly, the EGARCH and CSGARCH models, which also obtained good results, perform better than the standard GARCH model because they consider the asymmetric effects on the volatility of the cryptocurrency series. Considering that the conditional variance is not a constant process, where negative returns have a greater influence on the conditional variance of the series, they allow us to better capture the dynamics of the cryptocurrency.

In general, investors interested in predicting the future conditional variance of the series are advised to use the CSGARCH model, as this is how the periods of random shocks in the market operate and affect the future of the series. Future works shall include the comparison of GARCH-family models against other alternatives for variance modeling and forecasting, as well as the evaluation of forecasts in risk management applications such as in Value-at-Risk estimation.

# References

Satoshi Nakamoto. Bitcoin whitepaper. *URL: https://bitcoin.org/en/bitcoin-paper*, 2008.

Yakup Söylemez. Cryptocurrency derivatives: The case of bitcoin. In *Blockchain economics and financial market innovation*, pages 515–530. Springer, 2019.

Itai Agur, Anil Ari, and Giovanni Dell'Ariccia. Designing central bank digital currencies. *Journal of Monetary Economics*, 125:62–79, 2022.

Michael Kumhof and Clare Noone. Central bank digital currencies-design principles and balance sheet implications. 2018.

Darrell Duffie, Kelly Mathieson, and Darko Pilav. Central bank digital currency: Principles for technical implementation. *Available at SSRN*, 2021.

Andros Gregoriou. Cryptocurrencies and asset pricing. *Applied Economics Letters*, 26(12):995–998, 2019.

Shaen Corbet, Brian Lucey, Maurice Peat, and Samuel Vigne. Bitcoin futures—what use are they? *Economics Letters*, 172:23–27, 2018a.

Chung Baek and Matt Elbeck. Bitcoins as an investment or speculative vehicle? A first look. *Applied Economics Letters*, 22(1):30–34, 2015.

Florian Glaser, Kai Zimmermann, Martin Haferkorn, Moritz Christian Weber, and Michael Siering. Bitcoin-asset or currency? revealing users' hidden intentions. *Revealing Users' Hidden Intentions (April 15, 2014). ECIS*, 2014.

Eng-Tuck Cheah and John Fry. Speculative bubbles in bitcoin markets? an empirical investigation into the fundamental value of bitcoin. *Economics Letters*, 130:32–36, 2015.

Pedro Chaim and Marcio P Laurini. Is bitcoin a bubble? *Physica A: Statistical Mechanics and its Applications*, 517:222–232, 2019.

Shaen Corbet, Brian Lucey, and Larisa Yarovaya. Datestamping the bitcoin and ethereum bubbles. *Finance Research Letters*, 26:81–88, 2018b.

Benjamin M Blau. Price dynamics and speculative trading in bitcoin. *Research in International Business and Finance*, 41:493–499, 2017.

Dima Alberg, Haim Shalit, and Rami Yosef. Estimating stock market volatility using asymmetric garch models. *Applied Financial Economics*, 18(15): 1201–1208, 2008.

Juri Marcucci. Forecasting stock market volatility with regime-switching garch models. *Studies in Nonlinear Dynamics & Econometrics*, 9(4), 2005.

Reinaldo C Garcia, Javier Contreras, Marco Van Akkeren, and João Batista C Garcia. A garch forecasting model to predict day-ahead electricity prices. *IEEE transactions on power systems*, 20(2):867–874, 2005.

Paraskevi Katsiampa. Volatility estimation for bitcoin: A comparison of garch models. *Economics Letters*, 158:3–6, 2017.

Roy Cerqueti, Massimiliano Giacalone, and Raffaele Mattera. Skewed non-gaussian garch models for cryptocurrencies volatility modelling. *Information Sciences*, 527:1–26, 2020.

Yeguang Chi and Wenyan Hao. A horserace of volatility models for cryptocurrency: Evidence from bitcoin spot and option markets. *arXiv preprint arXiv:2010.07402*, 2020.

Pierre J Venter, Eben Mare, and Edson Pindza. Price discovery in the cryptocurrency option market: A univariate garch approach. *Cogent Economics & Finance*, 8(1):1803524, 2020.

Mohamed Fakhfekh and Ahmed Jeribi. Volatility dynamics of cryptocurrencies' returns: Evidence from asymmetric and long memory garch models. *Research in International Business and Finance*, 51:101075, 2020.

Shaen Corbet, Andrew Meegan, Charles Larkin, Brian Lucey, and Larisa Yarovaya. Exploring the dynamic relationships between cryptocurrencies and other financial assets. *Economics Letters*, 165:28–34, 2018c.

Andrew Phillip, Jennifer SK Chan, and Shelton Peiris. A new look at cryptocurrencies. *Economics Letters*, 163:6–9, 2018.

Henry L Gray, Nien-Fan Zhang, and Wayne A Woodward. On generalized fractional processes. *Journal of Time Series Analysis*, 10(3):233–257, 1989.

Stephen J Taylor. *Modelling financial time series*. World Scientific, 2008.

Donglian Ma and Hisashi Tanizaki. The day-of-the-week effect on bitcoin return and volatility. *Research in International Business and Finance*, 49: 127–136, 2019.

Harald Kinateder and Vassilios G Papavassiliou. Calendar effects in bitcoin returns and volatility. *Finance Research Letters*, page 101420, 2019.

Lawrence R Glosten, Ravi Jagannathan, and David E Runkle. On the relation between the expected value and the volatility of the nominal excess return on stocks. *The Journal of Finance*, 48(5):1779–1801, 1993.

M Ángeles López-Cabarcos, Ada M Pérez-Pico, Juan Piñeiro-Chousa, and Aleksandar Šević. Bitcoin volatility, stock market and investor sentiment. are they connected? *Finance Research Letters*, page 101399, 2019.

Wei Liu, Artur Semeyutin, Chi Keung Marco Lau, and Giray Gozgor. Forecasting value-at-risk of cryptocurrencies with riskmetrics type models. *Research in International Business and Finance*, 54:101259, 2020.

Richard Gerlach, Zudi Lu, and Hai Huang. Exponentially smoothing the skewed laplace distribution for value-at-risk forecasting. *Journal of Forecasting*, 32(6):534–550, 2013.

Paul Kupiec. Techniques for verifying the accuracy of risk measurement models. *The J. of Derivatives*, 3(2), 1995.

Peter F Christoffersen. Evaluating interval forecasts. *International economic review*, pages 841–862, 1998.

Robert F Engle and Simone Manganelli. Caviar: Conditional autoregressive value at risk by regression quantiles. *Journal of business & economic statistics*, 22(4):367–381, 2004.

Peter R Hansen, Asger Lunde, and James M Nason. The model confidence set. *Econometrica*, 79(2):453–497, 2011.

Paraskevi Katsiampa. An empirical investigation of volatility dynamics in the cryptocurrency market. *Research in International Business and Finance*, 50:322–335, 2019.

Gerrit Köchling, Philipp Schmidtke, and Peter N Posch. Volatility forecasting accuracy for bitcoin. *Economics Letters*, 191:108836, 2020.

Serkan Aras. Stacking hybrid garch models for forecasting bitcoin volatility. *Expert Systems with Applications*, 174:114747, 2021.

Yaohao Peng, Pedro Henrique Melo Albuquerque, Jader Martins Camboim de Sá, Ana Julia Akaishi Padula, and Mariana Rosa Montenegro. The best of two worlds: Forecasting high frequency volatility for cryptocurrencies and traditional currencies with support vector regression. *Expert Systems with Applications*, 97:177–192, 2018.

Tim Bollerslev. Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, 31(3):307–327, 1986.

Daniel B Nelson. Conditional heteroskedasticity in asset returns: A new approach. *Econometrica: Journal of the Econometric Society*, pages 347–370, 1991.

Robert F Engle, Halbert White, et al. *Cointegration, causality, and forecasting: a Festschrift in Honour of Clive WJ Granger*. Oxford University Press on Demand, 1999.

Andrew J Patton. Volatility forecast comparison using imperfect volatility proxies. *Journal of Econometrics*, 160(1):246–256, 2011.